A Recursive Algorithm for Analysis of Planar Multiple Lines on Composite Substrates for M(H)MIC's and High-Speed Interconnects

Yansheng Xu, Ke Wu, and Renato G. Bosisio

***Abstract*—A simple recursive algorithm is presented based on the method of lines for the analysis of multilayered multiple microstrip lines or slots. Our previously proposed scheme of vertical multi-subregion space discretization [1] is used to enhance the numerical accuracy. The recursive formulation is extended to model composite substrates which is aimed at reducing the unwanted coupling among different lines in M(H)MIC's and high-speed interconnects. Numerical results are shown for both quasistatic and hybrid-mode analyses. Results of multiple strips on a composite uniaxial anisotropic substrate are also presented.**

I. Introduction

There have been a number of documented works [2]–[7] on the analysis of multiple strips which have found widespread applications in various microwave integrated circuits (MIC's), monolithic MIC's, VLSI and MMIC interconnects. Nevertheless, the analysis methods become more complicated with a larger number of coupled strips. On the other hand, it is known that the field coupling or cross-talk as well as pulse distortion in high speed interconnects [2], [3] can be reduced through hollow segments between adjacent strips. Therefore, it is of both theoretical and practical importance to have both hybrid-mode and quasistatic methods which are able to promise accurate electromagnetic modeling of these complex structures in a simple and efficient way.

In our previous paper [1], a novel and efficient approach based on the method of lines [8]–[10] has been proposed to model single microstrip line on multilayered composite substrates with different segments. It is able to handle not only very narrow strips/very large slots topologies or vice versa but also simulate exactly bilaterally unbounded structures. In the following, this approach will be extended to model multiple strips or slots. To do so, field potentials are transferred from one strip to another and matched on the last one. This recursive procedure makes the algorithm very simple, flexible and easy to handle for a large class of complex planar composite structures. At the same time, the efficiency of the numerical calculation is enhanced significantly. It is especially convenient for various practical situations in which the number as well as the width of the strips are frequently modified from one calculation to another. In addition, it preserves all the advantages described in our previous paper [1]. This novel recursive algorithm is demonstrated through both quasistatic and hybrid-mode analyses for multiple coupled strips with hollow segments including uniaxial anisotropic substrates.

II. Theory

The analysis model is shown in Fig. 1 where multiple coupled microstrip lines are deposited on a bilaterally unbounded composite and segmented dielectric substrate. For simplicity, it is assumed that the structure is lossless and the thickness of strips is zero. Since analytical formulations of this approach have been well detailed in [1], our attention will be focused on those theoretical aspects related to multiple strip cases. The key point of the analysis is to derive the basic recursive formulation of the field potentials from one strip to another. The extension of this technique to structures with multiple slots is straightforward and will not be included for brevity.

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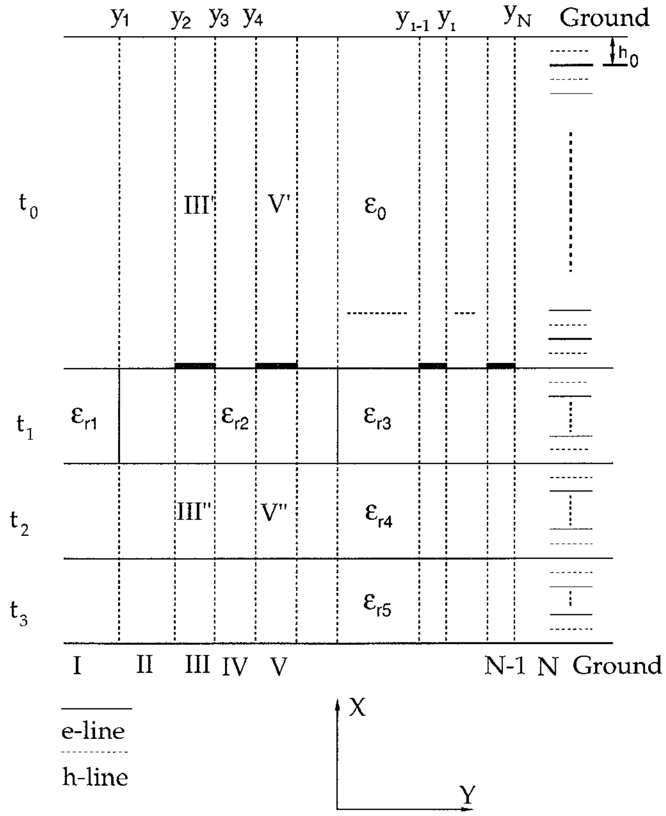

Fig. 1. Illustration of a bilaterally unbounded multiconductor planar transmission line deposited on a segmented multilayered substrate.

A. Quasi-Static Analysis

The whole structure is divided into subregions I, II, III,... and so on as shown in Fig. 1. The subregions are partitioned by vertical lines, incorporating either planar strips or vertical boundaries of different dielectric layers or both. The metallic strips thus become boundaries of additional subregions along the vertical direction for some related subregions, the subregion III is divided into III$'$ and III$''$ and so on (to name an example). As illustrated in Fig. 1, inhomogeneous layers along the x-direction are involved. Therefore, for the uniaxial anisotropic case, the electrical characteristics governed by the Laplace equation in terms of the electrical potential ψ can be expressed as

$$\frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial \psi}{\partial x}\right) + \varepsilon_y \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{1}$$

where ε_x and ε_y are dielectric permittivities along the x and y axis, respectively. As usual, the potential ψ is discretized and diagonalized using the appropriate transform matrix T [1], [8], [10] in each subregion. Subsequently, two different kinds of recursive formulations will be considered. One is related to subregions involving the metallic

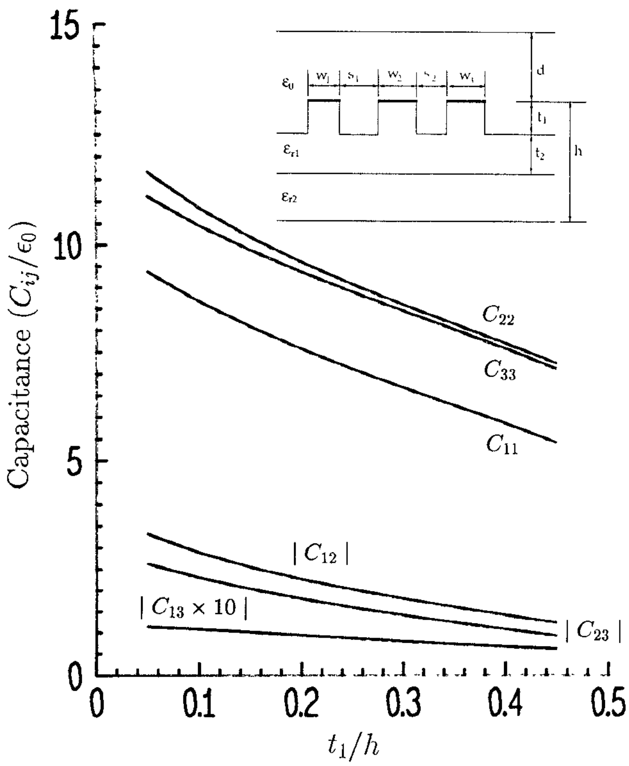

Fig. 2. Dependence of the capacitance matrix on the normalized height of the segmented layer t_1 of a three-conductor microstrip line with $\varepsilon_{r1} = 9.8$, $\varepsilon_{r2} = 2.3$, and $w_1 = 0.6$, $w_2 = 0.8$, $s_1 = 0.6$, $s_2 = 0.8$, $h = 1$, $d = 4$, $t_1 + t_2 = h/2$ (unit: mm).

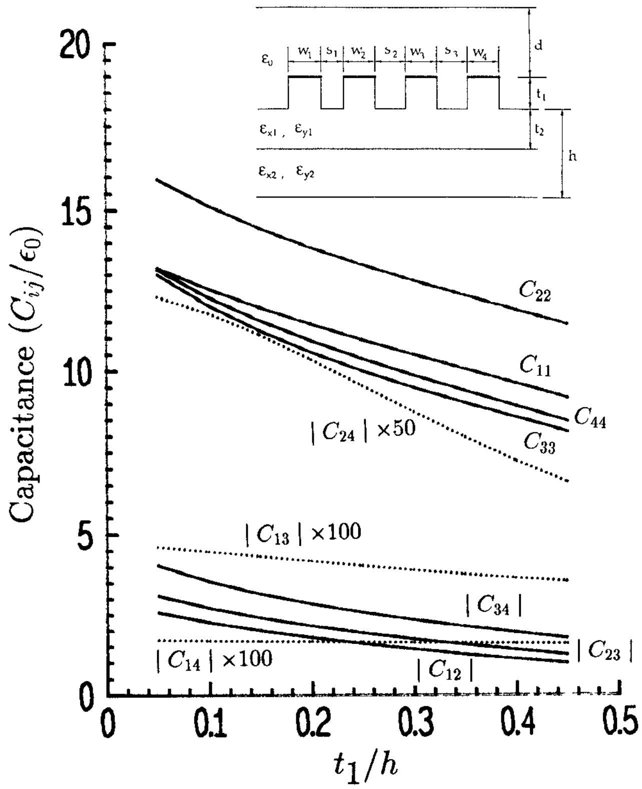

Fig. 3. Dependence of the capacitance matrix on the normalized height of the segmented layer t_1 of the uniaxial anisotropic substrate for a four-conductor microstrip line with $\varepsilon_{x1} = 11.6$, $\varepsilon_{y1} = 9.4$, $\varepsilon_{x2} = 3.4$, $\varepsilon_{y2} = 5.12$, and $w_1 = 1$, $w_2 = 1.2$, $w_3 = 0.6$, $w_4 = 0.8$, $s_1 = 0.8$, $s_2 = 0.6$, $s_3 = 0.4$, $h = 1$, $d = 4$, $t_1 + t_2 = h/2$ (unit: mm).

strip, such as the regions III′, III″, V′, V″,... in Fig. 1, and the remaining of the whole structure is subject to the other. For the first one, the subregion III (which is divided into III′ and III″) is taken as an example. The matrix relationship between potential $\vec{\psi}$ and its derivative is known at the left boundary ($y = y_2$), which can be expressed in the general form as follows:

$$\begin{bmatrix} \frac{d\vec{\psi}_{III'}(y_2)}{dy} \\ \frac{d\vec{\psi}_{III''}(y_2)}{dy} \end{bmatrix} = \|A\| \begin{bmatrix} \vec{\psi}_{III'}(y_2) \\ \vec{\psi}_{III''}(y_2) \end{bmatrix} + \vec{B} \tag{2}$$

where $\|A\|$ and $\vec{B}$ are the coefficient matrix and the coefficient vector [1], respectively. The transformed Laplace equation in subregions III′ and III″ becomes inhomogeneous due to the fact that the potential on the strip is not equal to zero. Solving inhomogeneous equations and transforming them back to the original domain lead to a hybrid relationship of the potentials and their derivatives between the left and right boundaries [1]

$$\begin{bmatrix} \frac{d\vec{\psi}_{III'}(y_2)}{dy} \\ \frac{d\vec{\psi}_{III''}(y_2)}{dy} \\ \frac{d\vec{\psi}_{III'}(y_3)}{dy} \\ \frac{d\vec{\psi}_{III''}(y_3)}{dy} \end{bmatrix} = \begin{bmatrix} -\gamma_{III'} & 0 & \alpha_{III'} & 0 \\ 0 & -\gamma_{III''} & 0 & \alpha_{III''} \\ -\alpha_{III'} & 0 & \gamma_{III'} & 0 \\ 0 & -\alpha_{III''} & 0 & \gamma_{III''} \end{bmatrix} \cdot \begin{bmatrix} \vec{\psi}_{III'}(y_2) \\ \vec{\psi}_{III''}(y_2) \\ \vec{\psi}_{III'}(y_3) \\ \vec{\psi}_{III''}(y_2) \end{bmatrix} + \begin{bmatrix} -\frac{(\alpha_{III'}-\gamma_{III'})\, T_{III'}\, \vec{p}}{\kappa^2_{III'}} \\ -\frac{(\alpha_{III''}-\gamma_{III''})\, T_{III''}\, \vec{q}}{\kappa^2_{III''}} \\ \frac{(\alpha_{III'}-\gamma_{III'})\, T_{III'}\, \vec{p}}{\kappa^2_{III'}} \\ \frac{(\alpha_{III''}-\gamma_{III''})\, T_{III''}\, \vec{q}}{\kappa^2_{III''}} \end{bmatrix} \tag{3}$$

where

$$\alpha_{III',III''} = T_{III',III''} \cdot \frac{\vec{\kappa}_{III',III''}}{\sinh(\vec{\kappa}_{III',III''} w)} T^{-1}_{III',III''},$$
$$\gamma_{III',III''} = T_{III',III''} \cdot \frac{\vec{\kappa}_{III',III''}}{\tanh(\vec{\kappa}_{III',III''} w)} T^{-1}_{III',III''},$$
$$\vec{p} = T^{-1}_{III'} \cdot [0, 0, \ldots, u_0]^t / h_0^2,$$
$$\vec{q} = T^{-1}_{III''} \cdot [u_0, 0, \ldots, 0]^t / h_0^2,$$
$$w = y_3 - y_2$$

and u_0 is supposed to be the potential on the metallic strip in subregion III. Eliminating $d\vec{\psi}(y_2)/dy$ and $\vec{\psi}(y_2)$yields

$$\begin{bmatrix} \frac{d\vec{\psi}_{III'}(y_3)}{dy} \\ \frac{d\vec{\psi}_{III''}(y_3)}{dy} \end{bmatrix} = \underbrace{\left\{ -\begin{bmatrix} \alpha_{III'} & 0 \\ 0 & \alpha_{III''} \end{bmatrix} \|Q\|^{-1} \begin{bmatrix} \alpha_{III'} & 0 \\ 0 & \alpha_{III''} \end{bmatrix} + \begin{bmatrix} \gamma_{III'} & 0 \\ 0 & \gamma_{III''} \end{bmatrix} \right\}}_{\|F\|} \times \begin{bmatrix} \vec{\psi}_{III'}(y_3) \\ \vec{\psi}_{III''}(y_3) \end{bmatrix} + \underbrace{\begin{bmatrix} \alpha_{III'} & 0 \\ 0 & \alpha_{III''} \end{bmatrix} \|Q\|^{-1} \left\{ \vec{B} + \begin{bmatrix} (\alpha_{III'} - \gamma_{III'})\vec{p}/\kappa^2_{III'} \\ (\alpha_{III''} - \gamma_{III''})\vec{q}/\kappa^2_{III''} \end{bmatrix} \right\}}_{\vec{G}} + \underbrace{\begin{bmatrix} (\alpha_{III'} - \gamma_{III'})\vec{p}/\kappa^2_{III'} \\ (\alpha_{III''} - \gamma_{III''})\vec{q}/\kappa^2_{III''} \end{bmatrix}}_{\vec{C}} \tag{4}$$

with

$$\|Q\| = \|A\| + \begin{bmatrix} \gamma_{\text{III}'} & 0 \\ 0 & \gamma_{\text{III}''} \end{bmatrix}.$$

In this way, the same relationship as (2) between the potential and its derivative is obtained at $y = y_3$ but with a different coefficient matrix $\|F\|$ and coefficient vector $\vec{G}$. Note that the order of the vectors $\vec{\psi}'_{\text{III}}(y)$, $\vec{\psi}''_{\text{III}}(y)$ is equal to n_1, n_2, respectively. These orders are determined by the number of discretized lines in the subregions III′ and III″. Hence, $\|F\|$ and $\|Q\|$ are $(n_1 + n_2) \times (n_1 + n_2)$ matrices.

A similar mathematical process in the subregion IV is demonstrated as an example for the situation without metallic strips. In this case, the transformed Laplace equation is homogeneous from which we obtain

$$\begin{bmatrix} \frac{d\vec{\psi}(y_3)}{dy} \\ \frac{d\vec{\psi}(y_4)}{dy} \end{bmatrix} = \begin{bmatrix} -\gamma & \alpha \\ -\alpha & \gamma \end{bmatrix} \begin{bmatrix} \vec{\psi}(y_3) \\ \vec{\psi}(y_4) \end{bmatrix} \tag{5}$$

which is similar to (3) except that u_0, $\vec{p}$ and $\vec{q}$ are all equal to zero and that this subregion is not split as in the former case since there is no strip embedded in this subregion. The relationship between the potential and its derivative at the left boundary $y = y_3$ is known as (4). To obtain the potential transfer function at the right boundary $y = y_4$, the matrices α and γ have to be split into submatrices as

$$\|\gamma\| = \begin{bmatrix} \gamma_{11}^{n_1\times n_1} & \gamma_{12}^{n_1\times 1} & \gamma_{13}^{n_1\times n_2} \\ \gamma_{21}^{1\times n_1} & \gamma_{22}^{1\times 1} & \gamma_{23}^{1\times n_2} \\ \gamma_{31}^{n_2\times n_1} & \gamma_{32}^{n_2\times 1} & \gamma_{33}^{n_2\times n_2} \end{bmatrix} \text{ and } \|\alpha\| = \begin{bmatrix} \alpha_{\text{I}}^{n_1\times n} \\ \alpha_0^{1\times n} \\ \alpha_{\text{II}}^{n_2\times n} \end{bmatrix}. \tag{6}$$

Through some matrix manipulations, another transfer function is obtained

$$\frac{d\vec{\psi}(y_4)}{dy} = \left[\|\gamma\| - \|\alpha\| \begin{bmatrix} C_{\text{I}}^{(n_1\times n)} \\ 0^{(1\times n)} \\ C_{\text{II}}^{(n_2\times n)} \end{bmatrix} \right] \vec{\psi}(y_4) + \|\alpha\| \begin{bmatrix} \vec{D}_{\text{I}}^{(n_1\times 1)} \\ u_0^{(1\times 1)} \\ \vec{D}_{\text{II}}^{(n_2\times 1)} \end{bmatrix} \tag{7}$$

where $n(n = n_1 + n_2 + 1)$ is the total number of lines in the subregion IV and $n_0 = n - 1$

$$\begin{bmatrix} C_{\text{I}}^{(n_1\times n)} \\ C_{\text{II}}^{(n_2\times n)} \end{bmatrix} = \|R\|^{(n-1)\times(n-1)} \begin{bmatrix} \alpha_{\text{I}}^{(n_1\times n)} \\ \alpha_{\text{II}}^{(n_2\times n)} \end{bmatrix}$$

and

$$\begin{bmatrix} \vec{D}_{\text{I}} \\ \vec{D}_{\text{II}} \end{bmatrix} = \|R\| \cdot \left[\vec{G} + u_0 \begin{bmatrix} \gamma_{12} \\ \gamma_{32} \end{bmatrix} \right]^{n_0\times 1} \tag{8}$$

with

$$\|R\| = \left(\|\vec{F}\| + \|\gamma'\|\right)^{-1} \text{and } \|\gamma'\| = \begin{bmatrix} \gamma_{11} & \gamma_{13} \\ \gamma_{31} & \gamma_{33} \end{bmatrix}.$$

The relationship between the potential and its derivative of the first subregion I and the last subregion N with the unbounded transverse section is readily obtained [1]. Such a recursive formulation may start from the subregion I, transferred from one strip to another, end up with the subregion N through the similar equations as (4) and (6) but in the relevant subregion. The deterministic matrix can be obtained at the boundary y_N, which is readily solved for the potential vector $\vec{\psi}(y_N)$ and relevant quantities. Subsequently, the potential vectors at each boundary $\vec{\psi}(y_i)$ $(i = 1,2 \cdots N-1)$ may be obtained by a reverse transfer. The charge distributions at each strip are then calculated together with the capacitance matrix of the strips [1]. Through this recursive algorithm, it can be seen that the dimension of the characteristic matrix remains always the same for arbitrary number of strips. This is a remarkable advantage over the conventional method of lines.

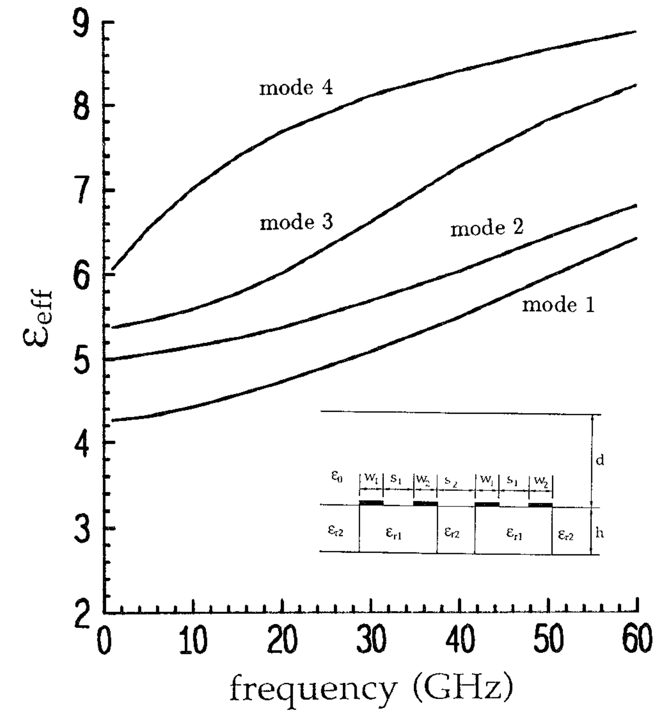

Fig. 4. Calculated dispersion characteristics of different modes for a four-conductor microstrip line with $w_1 = 0.6$, $w_2 = 0.4$, $s_1 = 0.4$, $s_2 = 0.5$, $h = 1$, $d = 4$, (unit: mm) $\varepsilon_{r1} = 9.8$, and $\varepsilon_{r2} = 4$.

B. Hybrid-Mode Analysis

The procedure used in the hybrid-mode analysis is quite similar to the quasistatic case except the handling of subregions involving metallic strips. Electromagnetic potentials ψ^e (LSM-x) and ψ^h (LSE-x) are governed by the Helmholtz and Sturm–Liouville differential equations [1], [8]–[10]. After discretization and transformation in the space domain, the ratio of the transformed electric and magnetic potentials and their derivatives at interface $y = y_i$ can be expressed through the same ratio at $y = y_{i-1}$. The tangential electric and magnetic fields can be obtained from Maxwell equations in the transformed domain at all interfaces. This has been detailed in [1], [8], [9]. After transforming back to the original domain, a field matrix relating magnetic field quantities to their electric counterparts can be derived at y_1. Similar to the quasistatic analysis, this matrix is then transferred along the y-direction to the interfaces, $y_2, y_3 \cdots y_N$ and we have

$$H_{N-1}|_{y_N} = U_{N-1} \cdot E_{N-1}|_{y_N}. \tag{9}$$

It should be pointed out that, in the transferring process through the subregions containing strips, the two complementary transfer matrices of each pair of subregions should be combined such as III′ and III″, V′, and V″ ... etc. into one single matrix, respectively. This implies a complete point matching of field components in the discrete space domain.

On the other hand, a similar field matrix U_N may be obtained for the subregion N which tends to infinity together with $y \to \infty$

$$H_N|_{y_N} = U_N \cdot E_N|_{y_N}. \quad (10)$$

Matching the tangential field components of subregions $N-1$ and N at $y = y_N$, a characteristic matrix is obtained and frequency-dependent propagation constants and related field parameters can be found by solving this determinant equation.

C. Numerical Results

To validate the proposed recursive algorithm, the characteristic impedance of even and odd modes of a coupled microstrip line is calculated by using the quasistatic modeling, showing a good agreement with [11]. Fig. 2 shows the capacitance matrix of a three-conductor microstrip line. Fig. 3 displays the capacitance matrix of a four-conductor microstrip line deposited on a segmented multilayer uniaxial anisotropic substrate. A typical CPU time is 5 min. for the calculation of this figure on a low-speed HP-400 workstation. The limiting line spacing used is around 300/mm. The calculated results change significantly with the normalized height of the segmented layer t_1. It is interesting that the self-capacitances C_{11}, C_{44}, and C_{33} tend to equal each other when the thickness t_1 approaches zero and their values diversify as t_1 becomes large. This can be explained by the fact that when $t_1 \to 0$, the coupling between the different strips increases drastically and the coupling effect makes C_{33} and C_{44} increase faster than C_{11} since the dimension s_3 is much less than s_1. As the thickness t_1 increases, the coupling effect diminishes and the difference between the self-capacitance become more pronounced.

Dispersion characteristics of a coupled microstrip line are also calculated. Fig. 3 shows dispersion characteristics of different modes of a microstrip line with four conductors on a segmented multilayered substrate.

III. Conclusion

This paper presents a recursive algorithm of the method of lines based on the vertical discretization [1] for the analysis of multiple strips or slots on composite multilayered substrates including uniaxial anisotropic materials. The advantage of this algorithm is that modeling on arbitrary multiple lines (or slots) is accomplished by a simple transferring process of the "standard" field matrices from one strip (or slot) to another. An additional identified advantage compared to the conventional method of lines is that the order of characteristic matrix remains always the same regardless of the number of strips or slots. This is more pronounced when a large number of strips or slots gets involved such as in high-speed interconnects. Our examples demonstrate potential application to a large class of planar circuits including complex composite substrates with hollow segments which were proposed to reduce the field coupling between different strips.

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The Propagation Constant of a Lossy Coaxial Line with a Thick Outer Conductor

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Abstract—**The microwave approximation for the propagation constant of a coaxial line becomes inaccurate below 1 MHz. An approximation is presented that is accurate over the entire operating frequency range of the line.**

I. Introduction

The propagation constant γ for the principal, transverse magnetic mode on a lossy coaxial line has been known for many years [1] and appears in the field equations with the form

$$\boldsymbol{F} = \boldsymbol{F}_0 e^{j\omega t - \gamma z} \quad (1)$$

where $\boldsymbol{F}$ represents any one of the principal mode field components, ω the radian frequency, t the time, and z the axial distance along the line.

An exact calculation of γ is complicated by the need to solve the coaxial line determinantal equation involving Bessel functions of the first and second kinds with complex arguments. Furthermore, the only approximation for γ in common usage today is that originally derived by Stratton [1], a first-order perturbation equation in the

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